General Rules

These are general guidelines for the grading of all problems. All of the rules listed below refer to deductions on specific items in the marking scheme.

Using absurd physical or mathematical concepts (despite correct answers)	No points given
Giving correct answer but the method is unclear	Deduct 50% of the marks for final answer
Minor mistakes in the calculations, e.g., wrong signs, numerical constants	Deduct 20% of the marks for that part, unless there is a physical incoherence
Too many or too few significant figures ($n \pm 1$ where n is the expected number of significant figures)	Deduct 50% of the marks for final answer
Correct answer but missing or wrong units	No points given for final answer
For numerical values, the answer is not within a 5% precision range	No points given for final answer, unless differently stated in the problem or marking scheme
Error resulting from an earlier part error, for which the student already lost marks, if the answer is physically reasonable	No double penalization, full points given
Error resulting from an earlier part error, if the answer is physically unreasonable	No points given

First example: If due to an error in an earlier item, the student calculates the speed of a satellite as 4.0×10^4 m/s instead of 4.5×10^4 m/s, they will *only* lose marks for the earlier item. However, if, for the same reason, a student calculates the velocity of the satellite as 4.5×10^{12} m/s, they should realize this is wrong (higher than the speed of light) and thus should lose some marks for this item as well.

Second example: Consider a question that requires a student to determine the temperature of a planet that orbits a star with luminosity L at a distance r. To answer correctly, the student must calculate the star's radiant flux, the power incident on the planet, the power radiated by the planet, and then, using the condition of thermal equilibrium, the planet's temperature. If the student only calculates the radiant flux and arrives at the correct final answer without *clearly* explaining the other steps, they will receive marks *only* for the flux calculation, but not for the other steps, neither for the final answer.

T1. Sundial (10 points)

- **2 points** will be awarded for each correct answer.

T2. Galaxy Cluster (10 points)

- **2 points**: correctly making use of the Virial Theorem to relate U and K.
  Deduction of **-0.5 points** if the student uses that $|U| = K$.
- **2 points**: obtain a correct expression for K in terms of M_T and σ_v.
  Students who only write the definition of kinetic energy as a sum for all galaxies without relating it to the velocity dispersion will only receive **0.5 points**.
- **2 points**: obtain a correct expression for U in terms of M_T, R and G.
  Full marks are awarded as long as the student uses an *acceptable method of estimation.* For U in the form $-\alpha G M_T^2/R$, we suggest answers with factors $\alpha = [1/2, 3/5, 1]$ are considered correct, provided they use a valid method. Valid methods include, but are not necessarily limited to: dimensional analysis ($\alpha = 1$); homogeneous sphere ($\alpha = 3/5$); $\langle r_{ij}^{-1} \rangle = R^{-1}$ approach ($\alpha = 1/2$).
- **If the student writes right away that $M_T \sim \frac{R\sigma_v^2}{G}$ (For instance, using dimensional analysis or using circular orbits)**, award **2.5 points out of the 6 points totaled by the first three criteria.**
- **1 point**: writing $\sigma_{v_r} = c\sigma_z$.
  It is fine if the student just assumes that $\sigma_{v_r} = \sigma_v$, or does not realize that σ_z is directly related to σ_{v_r}, not σ_v.
- **1 point**: correctly calculating the physical size (radius or diameter) of the cluster using D_A and $\Delta\theta$. Numerical value not needed.
- **2 points**: correct estimate for the mass. Regardless of the method, answers inside the range $3 \times 10^{13} M_\odot \leqslant M_T \leqslant 3 \times 10^{14} M_\odot$ obtain **full marks**. **No points given** for this criterion if outside the range.

T3. Asteroid (10 points)

(a)

- **2 points**: correctly drawing or describing the geometry of the elliptical orbit of the asteroid. Students who identify that the trajectory can be approximated with a degenerate ellipse but do not show a correct understanding of the geometry of the ellipse will receive **1 point**.
- **2 points**: using either the principle of conservation of energy or the vis-viva equation to determine that $d = a$. Students who set up a correct equation but get a wrong result will receive **1 point**.
- **1 point**: using Kepler's third law to find an expression for the period.
- **2 points**: using Kepler's second law to calculate the interval. Students who use Kepler's second law but set up the equation incorrectly will receive **1 point**.
- **1 point**: correct final answer.

(b)

- **1.5 point**: using Kepler's second law to calculate the interval. It is also acceptable to subtract the interval calculated on the previous item from the total period.
- **0.5 point**: correct final answer.

T4. White Dwarf (10 points)

(a)
- **1.5 points**: using hydrostatic equilibrium with linear pressure gradient to find P_c in terms of M, R and constants.
- **1.5 points**: relating ρ and n_e through $\rho = \mu_e m_p n_e$
- **1.5 points**: algebraically manipulating given equation of state correctly and previous results to find a in terms of constants and μ_e.
- **1.5 points**: algebraically manipulating given equation of state correctly and previous results to find that $b = -1/3$.

(b)
- **1 point**: noticing $\mu_e \approx 2$.
- **3 points**: using found expression and obtaining correct final answer.

T5. CMB (10 points)

(a)
- **1 point**: using Wien's displacement law to find λ_{today}.
- **1 point**: applying the definition of redshift to establish a relation between λ_{today} and $\lambda_{infrared}$.
- **1 point**: correct final answer.

(b)
- **1 point**: applying the definition of the scale factor to obtain a_{ir}.
If the student just writes $a = 1/(1+z)$ but does not calculate a_{ir}, deduct **-0.5 points**.
- **1 points**: expressing da/dt in terms of $H(t)$ and $a(t)$.
- **2 points**: Using the Friedmann Equation in a matter dominated Universe to find the relation $H^2(t) = \dfrac{H_0^2}{a^3(t)}$ between the Hubble parameter $H(t)$ and the scale factor $a(t)$.
- **2 points**: manipulating the previous expression and integrating to find a formula for the age of the universe at the mentioned epoch.
- **The student will receive 3 points, 1 point from the second item of (b) and 2 points from the fourth item, if they just quote that $t \propto a^{3/2}$ for a matter-dominated Universe.**
- **1 point:** correct final answer.

T6. Cluster Photography (20 points)

(a)
- **1 point**: correctly defining the concept of plate scale, even if directly with the formula.
- **0.5 point**: correctly calculating the focal length of the telescope by using the focal ratio. If the expression is directly used in the formula for the plate scale, points should still be awarded.
- **1.5 points**: correct final answer.

(b)
- **1 point**: calculating the diameter of the image in the focal plane using the plate scale. If this step is implicit within the estimation of the number of pixels covered by the image, points should still be awarded.
- **2 points**: estimating the number of pixels covered by the image using a consistent method. The two suggested approaches are: estimate the area of the image and divide by the area of a single pixel or estimate the number of pixels corresponding to a diameter. However, if any other plausible method is developed, full credits should be awarded.
- **1 point**: correct final answer within the specified range. If the student uses only 2 significant figures, include the acceptable values of 15000 and 16000 pixels.

(c)

- **3 points**: correctly defining the signal-to-noise ratio formula. If the student neglects the term associated with Dark Noise with valid arguments, full credits should be awarded.
- **2 points**: calculating the photon counts associated with CCD noise (1 point for each term). If the term related to Dark Noise was neglected in the previous step - with plausible argumentation, the 1 point from calculating σ_{DN} should still be given.
- **1 point**: calculating the solid angle subtended by the globular cluster.
- **1 point**: calculating the apparent magnitude of the cluster in the V-band.
- **1 point**: calculating the photon flux of the globular cluster. If the **correct** expression for the photon flux is replaced directly in the calculation of the source count (next step), full credits should still be awarded.
- **1 point**: calculating the photon counts associated with the globular cluster. 0.5 point should be given for the correct expression of the source count even if the calculated value for N_{GC} is incorrect.
- **1 point**: using the signal-to-noise ratio for estimating the sky count.
- **1 point**: applying the inverse procedure to estimate the flux of photons from the sky.
- **1 point**: applying the inverse procedure to estimate the magnitude of the sky.
- **1 point**: applying the inverse procedure to estimate the surface brightness of the sky. If the final value does not fall within the given range, deduct 0.5 point.

T7. Castaway (20 points)

(a)

- **2 points**: deriving an expression for the latitude as a function of the declination and the azimuth. Note that the students don't necessarily need to derive an expression for the azimuth during the sunrise and the sunset. They should also get full marks on this criterion if they simply write down the expression for the azimuth and proceed from there.
- **2 points**: determining the correct upper bound for the latitude of the island. Students who identify that the upper bound corresponds to the Southernmost declination of the Sun but fail to determine the latitude will receive **1 point**.
- **3 points**: determining the correct lower bound for the latitude of the island. Students who identify that the lower bound corresponds to a $0°$ declination but fail to determine the latitude will receive **1.5 point**.

(b)

- **2 points**: noticing that the ratio between the sine of the declination and the cosine of the azimuth remains constant for a given latitude.
- **1 point**: correctly drawing or describing the geometry of the situation.
- **1 point**: finding an expression for the declination on day 40 as a function of θ on day 40 and the obliquity of the ecliptic.
- **2 points**: finding an expression for θ on day 0 as a function of θ on day 40.
- **2 points**: finding a second expression that relates θ on day 0 and θ on day 40.
- **3 points**: combining the equations to find the value of θ_0.
- **1 point**: finding the value of δ_0
- **1 point**: correct final answer.

T8. Binary Hardening (25 points)

(a) **For Solution I - Directly using energy-momentum conservation:**

- **2 points**: correctly writing down conservation of mechanical energy for the star.

- **2 points**: correctly writing down conservation of angular momentum for the star.

For Solution II - Exploring the geometry of the hyperbolic orbit:

- **2 points:** writing an expression that relates the eccentricity of the orbit to v and b (from the angular momentum).
- **1 point:** correctly relating v to the semi-major axis of the hyperbolic orbit (using the total mechanical energy).
- **1 point:** correctly relating the periastron distance r_p to the eccentricity e.

For either solution:

- **1 point:** Solving for b and correct final answer.
  No points deducted if the final answer is not in approximate form $b \approx \frac{\sqrt{2GMa}}{v}$ as in the solution.

(b) **For Solution I - Energy and momentum conservation in the CM frame**

- **1.5 point**: correctly writing down conservation of linear momentum for the star + component.
- **1.5 point**: correctly writing down conservation of mechanical energy for the star + component.
- **1 point**: using that $m \ll M$ to simplify the answer.

For Solution II - Simpler solution in the frame of a component:

- **1.5 point**: noticing that the star approaches a component with velocity $-\vec{V}$ in this frame.
- **1.5 point**: noticing that the star bounces back with a final speed $\vec{V}$ in this frame.
- **1 point**: return to the frame of the CM by adding $\vec{V}$ to the velocity calculated.

For either solution:

- **1 point**: calculating orbital velocity of the component.
- **1 point**: correct final answer for $v_{\rm f}$.

(c)
- **1 point** calculating energy ΔK_* acquired by a star during an encounter with the binary.
- **4 points:** finding number of stars dN in the annular cylinder that interact with the star during time dt: $2\pi n\, b\, v\, \mathrm{d}b\, \mathrm{d}t$.
  If the student considers that the binary moves with the same speed for all stars in the frame of a far away star (say, v_0) and writes $dN = \pi(b_{max}^2 - b_{min}^2)v_0 dt$, deduct **-2 points**.
- **1 point:** using answer from (a) to write dϕ in terms of db and given data in order to eliminate v dependence.
- **3 points:** summing/integrating dϕ over the given range of impact parameters to obtain the total rate of encounters ϕ.
- **2 points** correctly writing dE_{bin}/dt (or just dE_{bin}) using ΔK_* and ϕ.
- **2 points:** correctly writing dE_{bin}/dt (or just dE_{bin}) using the total energy of the binary.
- **1 point:** equating the two to find $H = 8\pi$.

T9. Physics of Accretion (35 points)

(a)
- **2 points**: evaluating the centripetal force expression to calculate the velocity v_k.
- **2 points**: obtaining the correct expression for the total energy.

- **1 point**: using the fact that $r \ll R_{max}$ to simplify the expression for $E(R_{max}) - E(r)$.
- **1 point**: correct final expression.

(b)
- **3 points**: obtaining an expression for L_{tot}.
- **2 points**: correct final value.

(c)
- **5 points**: evaluating the expression for energy using reasonable approximations, such as the binomial approximation.
- **3 points**: correct final expression.

(d)
- **4 points**: obtaining the expression for the area of the ring. **2 points** should be deducted if the student misses the factor 2.
- **2 points**: using Stefan-Boltzmann's Law to relate temperature and area.
- **2 points**: writing the thermodynamic equilibrium expression.
- **2 points**: correct final expression.

(e)
- **1.5 points**: correctly calculating the luminosity.
- **1.5 points**: correctly calculating the wavelength.

(f)
- **1 point**: correctly calculating the temperature.
- **2 points**: correctly calculating the mass.

T10. Greatest Eclipse (75 points)

(a)
- **1 point**: understanding that the declination is the complement of the polar angle.
- **1 point**: calculating the declination of the Moon.
- **1 point**: calculating the declination of the Sun.

(b)
- **1 point**: understanding that the right ascension is the sum of the azimuthal angle and the local sidereal time at Greenwich.
- **1 point**: calculating the right ascension of the Moon.
- **1 point**: calculating the right ascension of the Sun.

(c)
- **1 point**: understanding that the vector corresponds to the subtraction of the Cartesian coordinates of the Sun from the Cartesian coordinates of the Moon.
- **1 point**: correct vector.
- **1 point**: correct modulus.
- **1 point**: correct unit vector.

(d)
- **5 points**: understanding that adding the vector position of the Moon with a product of the unit vector by a constant will result in the position vector of the intersection between the axis of the Moon's shadow cone and the surface of the Earth, whose module equals the center of the Earth.
- **3 points**: finding an expression for the constant factor k.
- **1 point**: correctly determining the numerical value of k.
- **2 points**: writing down the position vector of the intersection between the axis of the Moon's shadow cone and the surface of the Earth.
- **2 points**: finding the spherical coordinates of the intersection between the axis of the Moon's shadow cone and the surface of the Earth.
- **2 points**: finding the geographic coordinates of the intersection between the axis of the Moon's shadow cone and the surface of the Earth.

(e)
- **3 points**: demonstrating a correct understanding of the geometry of the situation. Note that it is also acceptable to consider that the angle β is roughly equal to 90°. Using this alternative approximate geometry, the angles marked as right angles in the official solution would actually be acute angles and their numerical values would need to be calculated.

- **2 points**: setting up a correct equation to calculate the angle β.
- **1 point**: correctly calculating the value of the angle β.
- **3 points**: setting up a correct equation to calculate the radius of the umbra.
- **1 point**: correctly calculating the value of the radius of the umbra.

(f)
- **2 points**: setting up a correct equation to calculate the velocity of the Earth's rotation at the latitude of the center of the umbra.
- **1 point**: correctly calculating velocity of the Earth's rotation at the latitude of the center of the umbra.

(g)
- **2 points**: setting up a correct equation to calculate the velocity of the Moon. **1 point** should be deducted if the student assumes a circular orbit.
- **2 points**: correctly calculating velocity of the Moon.

(h)
- **6 points**: correctly drawing or describing the geometry of the situation.
- **3 points**: setting up a correct equation to calculate κ.
- **2 points**: determining the value of κ.
- **3 points**: correct velocity vector. **1 point** per component.

(i)
- **3 points**: understanding that it is possible to obtain system III by rotating system II by the difference in polar angles
- **6 points**: correct algebraic expressions for the components of the vector in system III. **2 points** per component.
- **1 point**: correct numerical values of the components of the vector. **1/3 of a point** per component.

(j)
- **2 points**: correctly determining the x component of the vector.
- **1 point**: correctly determining the y component of the vector.
- **1 point**: correctly determining the z component of the vector. It is also acceptable to say that the z-component of the vector is equal to the z component of the velocity of the Moon on system III, as long as the student understands that only the x and y components of the vector are relevant to calculate the modulus of the velocity of the umbra on the surface of the Earth.
- **1 point**: correctly setting up an equation to calculate the modulus of the velocity of the umbra.
- **1 point**: calculating the modulus of the velocity of the umbra.

(k)
- **2 points**: understanding that the duration of the totality can be approximated as the ratio of the diameter of the umbra and the velocity of the umbra.
- **1 point**: correctly estimating the duration of the totality.

T11. Ground Tracks (75 points)

(a)
- **1 point**: identifying, by any means, that the nodal precession rate is 360° per year. This could be in a written form, image or directly with a formula.
- **2 points**: correct final result. **-1 point** if the student writes a negative value.

(b)
- **2 points**: obtaining the expression for the difference in longitude as a function of the orbital period.
- **2 points**: correct final result for the period. Note that the values of λ selected by the students may vary.
- **2 point**: correctly identifying in the graph the value of maximum latitude.
- **2 points**: taking the supplementary angle of the maximum latitude as the inclination.

(c)
- **2 points**: correctly calculating the semi-major axis.

(d)
- **1 point**: correctly calculating the number of orbits.

(e)
- **2 points**: correctly finding the relation of θ and ϕ.
- **3 points**: Correctly equating that the total angle travelled by the satellite depends on the number of whole orbits n, θ_0 and θ_M. It is also possible to equate using θ_C and different θ_0, using $\theta_C - \theta_o$, i.e., doing the inverse procedure of the solution.
- **1 point**: Correctly considering $n = 3$. (it is also possible to consider $n = 2$ if the previous equation had the signal of the angles swapped.
- **1 point**: Correctly equating Δt with $\Delta\theta$.
- **3 points**: Correctly equating the time T_C in function of Δt and $\lambda_C - \lambda_M$. **-1.5 points** if the student didn't correct local time for longitude.
- **1 point**: correct final answer.

(f)
- **1 point**: Correctly estimating i through the graph
- **1 point**: Correct value for T using argument of geosynchronous orbit. If the student directly writes the (well-known) values for a period and semi-major axis for the geosynchronous orbit without any calculation, all credits should still be given.
- **2 points**: Correct value for a using Kepler's 3rd law.

(g)
- **2 points**: understanding that the elapsed time to be calculated is the movement between the semi-latus rectum.
- **2 points**: Correctly equating Areas of the equivalent circle for Kepler 2nd law. Alternative: obtaining the eccentric anomaly at the semi-latus rectum as a function of the eccentricity.
- **4 points**: Correctly multiplying area of circle by (b/a) and applying correct simplifications. Alternative: obtaining the mean anomaly as a function of the eccentricity.
- **2 points**: Correctly arriving at the formula for the area of apogee. Alternative: obtaining an expression for the elapsed time t as a function of the eccentricity.
- **2 points**: Correctly applying Kepler 2nd law. Alternative: obtaining the ratio T'/T.

(h)
- **2 points**: Correctly relating the T' and $\Delta\lambda$ using the semi-latus rectum
- **2 points**: Correctly simplifying previous equation to get it in function of $\Delta\lambda$ and e.
- **2 points**: Correct value for $\Delta\lambda$ measured from the graph. **-1 point** if $\Delta\lambda$ has the same absolute value but positive.
- **3 points**: Applying correct approximations or solving Equation using iteration.
- **1 points**: Correct numerical value for e. It is only necessary one significant figure.

(i)
- **2 points**: correctly identifying the condition for a retrograde motion to occur - $\omega_{RA} = \omega_{\oplus}$.
- **2 points**: obtaining formulas to relate α and δ to i and the true anomaly.
- **2 points**: calculating the expression for the projected velocity of the satellite. If the student decomposes the velocity vector directly and the final result is correct, the 2 points of the previous step should be awarded.
- **2 points**: correct expression for $\dot{\alpha}$. If the student uses equivalent expressions that lead to the same results, the 2 points should still be given.
- **2 points**: using conservation of angular momentum to obtain $\dot{\theta}$.
- **2 points**: substituting the expression for the polar radius.
- **2 points**: correct final equation.
- **2 points**: correctly reading the value of δ from the graph.
- **2 points**: correctly calculating the values of θ.

(j)

- **2 points**: Correctly reasoning that, in the limiting case, the point of inversion will occur in the aphelion.
- **1 point**: $\delta = i$ when satellite is in its northernmost position.
- **2 points**: Correctly substitute values in the previous item formula
- **1 point**: correctly calculating the numerical value using an iteration method.